Universidad de Costa Rica
Escuela de Matemática
Proyecto MATEM 2012

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SOLUCIÓN IV EXAMEN PARCIAL CÁLCULO I

1. Calcule las siguientes integrales:

(a) $\int \operatorname{sen}(\sqrt{x})\ dx$ *(6 puntos)*

Sea $y^2 = x \Rightarrow 2y\,dy = dx.$ Entonces se tiene

$$\int \sin(y)2y\ dy = 2\int \sin(y)y\ dy$$

Consideremos $u = y \Rightarrow du = dy$ y $dv = \sin(y)\ dy \Rightarrow v = -\cos(y)$

$$-2y\cos(y) + 2\int \cos(y)\ du = -2y\cos(y) + 2\sin(y) + C$$

$$= -2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right) + C.$$

(b) $\int \frac{1}{\left(5-4x-x^2\right)^{\frac{5}{2}}}\ dx$ *(8 puntos)*

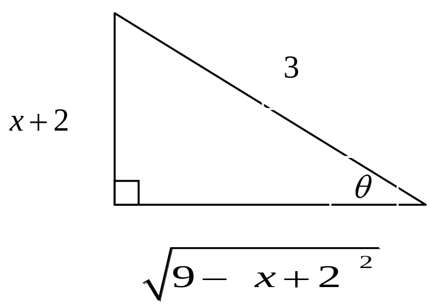
$$5-\left(x^2+4x+4\right)+4 = 9-\left(x+2\right)^2$$

$$\int \frac{1}{\left[9-\left(x+2\right)^2\right]^{\frac{5}{2}}}dx. \quad \text{Sea } x+2 = 3\sin\theta \Rightarrow dx = 3\cos\theta\ d\theta$$

$$= \int \frac{3\cos\theta\ d\theta}{\left(\sqrt{9\cos^2\theta}\right)^5} = \int \frac{3\cos\theta\ d\theta}{3^5\cos^5\theta} = \frac{1}{3^4}\int \frac{1}{\cos^4\theta}\ d\theta = \frac{1}{3^4}\int \sec^4\theta\ d\theta$$

$$= \frac{1}{3^4}\int \sec^2\theta\left(1+\tan^2\theta\right)\ d\theta = \frac{1}{3^4}\int \sec^2\theta d\theta + \frac{1}{3^4}\int \sec^2\theta\tan^2\theta\ d\theta$$

$$= \frac{1}{3^4}\tan\theta + \frac{1}{3^4}\frac{\tan^3\theta}{3} + C.$$



Regresando a la variable "x". Finalmente se tiene:

$$I = \frac{x+2}{3^4\sqrt{9-\left(x+2\right)^2}} + \frac{\left(x+2\right)^3}{3^5\left[\sqrt{9-\left(x+2\right)^2}\right]^3} + C$$

(c) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{1}{3+2\cos(x)}\,dx$ *(5 puntos)*

$$Sea\ u=\tan\left(\frac{x}{2}\right)\Rightarrow \frac{2}{1+u^2}\,du=dx\ \ y\ \ \cos(x)=\frac{1-u^2}{1+u^2}$$

Trabajando con la integral indefinida:

$$\int\frac{\frac{2}{1+u^2}\,du}{3+2\left(\frac{1-u^2}{1+u^2}\right)}=\int\frac{\frac{2}{1+u^2}}{\frac{3(1+u^2)+2(1-u^2)}{1+u^2}}\,du=\int\frac{\frac{2}{1+u^2}}{\frac{5+u^2}{1+u}}\,du=2\int\frac{1}{5+u^2}\,du=\frac{2}{\sqrt{5}}\arctan\left(\frac{u}{\sqrt{u}}\right)+C$$

$$\text{Luego}\quad \int_0^{\frac{\pi}{2}}\frac{1}{3+2\cos(x)}\,dx=\left[\frac{2}{\sqrt{5}}\arctan\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{5}}\right)\right]_0^{\frac{\pi}{2}}=$$

$$\frac{2}{\sqrt{5}}\arctan\left(\frac{\tan\left(\frac{\pi}{4}\right)}{\sqrt{5}}\right)-\frac{2}{\sqrt{5}}\arctan\left(\frac{\tan(0)}{\sqrt{5}}\right)=\frac{2}{\sqrt{5}}\arctan\left(\frac{1}{\sqrt{5}}\right)-0=\frac{2}{\sqrt{5}}\arctan\left(\frac{1}{\sqrt{5}}\right)$$

(d) $\displaystyle\int\frac{x^3-15x^2-23x-75}{(x^2+5)(x^2-9)}\,dx$ *(6 puntos)*

Haciendo la descomposición en fracciones parciales se tiene:

$$\frac{x^3-15x^2-23x-75}{(x^2+5)(x-3)(x+3)}=\frac{A}{x-3}+\frac{B}{x+3}+\frac{Cx+D}{x^2+5}$$

$$\Rightarrow\frac{A(x+3)(x^2+5)+B(x^2+5)(x-3)+(Cx+D)(x^2-9)}{(x^2+5)(x-3)(x+3)}$$

$$\Rightarrow x^3-15x^2-23x-75=A(x+3)(x^2+5)+B(x^2+5)(x-3)+(Cx+D)(x^2-9)$$

- $x=3\Rightarrow -252=A(6)(14)\Rightarrow -3=A$
- $x=-3\Rightarrow -168=B(-6)(14)\Rightarrow 2=B$
- $x=0\Rightarrow -75=-3(3)(5)+2(-3)(5)+D(-9)\Rightarrow -75=-75+D(-9)\Rightarrow 0=D$
- $x=1\Rightarrow -112=-3(4)(6)+2(-2)(6)+C(-8)\Rightarrow -16=-8C\Rightarrow 2=C$

Luego:

$$I=\int\frac{-3}{x-3}+\frac{2}{x+3}+\frac{2x}{x^2+5}\,dx=-3\ln|x-3|+2\ln|x+3|+\ln\left(x^2+5\right)C$$

(e) $\int_0^4 |2x+3| \ dx.$ *(5 puntos)*

Sabemos que $|2x+3| = \begin{cases} 2x+3 & si \quad x \geq \frac{-3}{2} \\ -(2x+3) & si \quad x < \frac{-3}{2} \end{cases}$. Entonces

$$\int_0^4 (2x+3)\, dx = \frac{2x^2}{2} + 3x\Big|_0^4 = x^2 + 3x\Big|_0^4 = \left[4^2 + 3\cdot\ 4\ \right] - \ 0^2 + 3\cdot 0 \ = 16 + 12 = 28.$$

2. Clasifique las siguientes integrales según su especie. Determine si son convergentes o divergentes calculando sus respectivos límites.

(a) $\int_{-\infty}^{0} xe^x \ dx$ *(7 puntos)*

La integral es de primera especie

$\lim\limits_{K\to-\infty} \int_k^0 xe^x dx$. Aplicamos integración por partes:

$u = x \Rightarrow du = dx$ y $dv = e^x dx \Rightarrow v = e^x$

$$\lim_{K\to-\infty}\left[\ x\cdot e^x\ \Big|_K^0 - \int_K^0 e^x dx\right] = \lim_{k\to-\infty}\left[\ x\cdot e^x\ \Big|_K^0 - e^x\Big|_K^0\right] =$$

$$\lim_{K\to-\infty}\left[\ 0\cdot e^0 - ke^K\ \ - \ e^0 - e^K\ \right] = \lim_{K\to-\infty}\left[-ke^K\ -1 + e^k\right] = \lim_{K\to-\infty} - Ke^K + \lim_{K\to-\infty} e^K\ -1 =$$

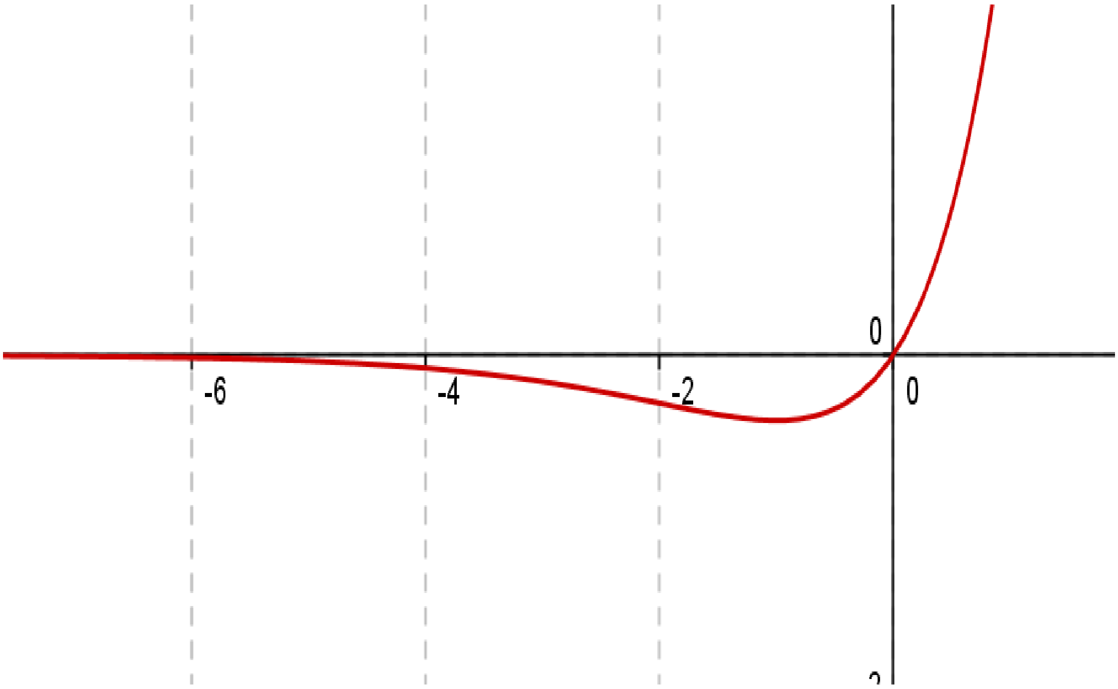
$$\underset{FI}{\infty\cdot 0} + -1$$

$$\lim_{K\to-\infty} \frac{-K}{e^{-K}} = \frac{\infty}{\infty} \text{ Aplicando L'Hopital}$$

$\lim\limits_{K\to-\infty} \frac{-1}{e^{-K}\cdot -1} = \lim\limits_{K\to-\infty} \frac{1}{e^{-K}} = 0$. Entonces se tiene que:

$$\lim_{K\to-\infty} - Ke^K + \lim_{K\to-\infty} e^K\ -1 = 0 + -1 = -1$$

$\therefore \int_{-\infty}^{0} xe^x dx$ converge.



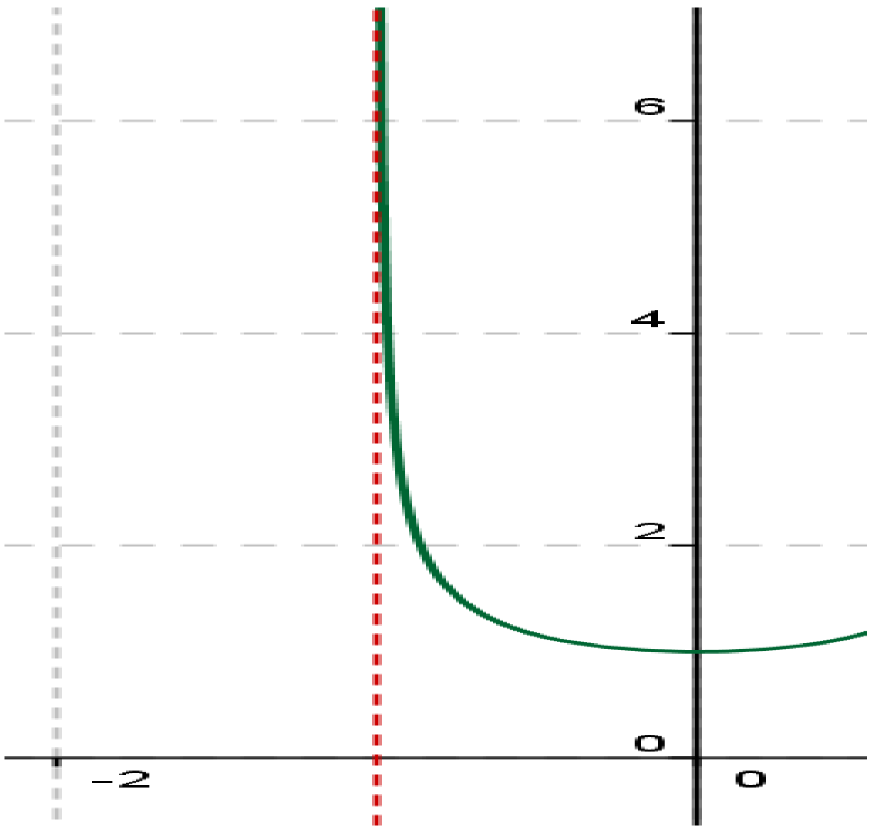
(b) $\int_{-1}^{0} \frac{1}{\sqrt{1-x^2}}\, dx$ *(7 puntos)*

La integral es de segunda especie

$$\lim_{k\to 0^+} \int_{-1+k}^{0} \frac{1}{\sqrt{1-x^2}}\, dx = \lim_{k\to 0^+} \arcsin\ \ x\ \Big|_{-1+k}^{0} =$$

$$\lim_{k\to 0^+}\left[\arcsin\ \ 0\ \ -\arcsin\ \ -1+k\ \ \right] = \lim_{k\to 0^+} 0-\arcsin\ \ -1+k\ \ = 0-\left(\frac{\pi}{2}\right) = 0+\frac{\pi}{2} = \frac{\pi}{2}$$

$$\therefore \quad \int_{-1}^{0} \frac{1}{\sqrt{1-x^2}}\, dx \text{ converge.}$$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